Appendix 3

Further discussions on Identification Results

In Appendix 2 we got a set of symbolic solutions to all the parameters in the enhanced model. The results are repeated here,

$$r_1 = \frac{n_1 Z_{11b} + Z_{11a}\omega n_2 + \omega n_3 Z_{12a} + Z_{12b}}{\omega n_2} \tag{1}$$

$$r_2 = \frac{n_1 Z_{11b} + Z_{22a}\omega n_3 + \omega n_2 Z_{12a} + Z_{22b}}{\omega n_2} \tag{2}$$

$$g_2 = -\frac{\omega n_2}{(n_1)^2 Z_{11b} + n_1 n_3 Z_{12a}\omega + n_1 Z_{12b} - n_2 Z_{12a}\omega + n_2 n_3 Z_{12b}(\omega)^2 + (n_2)^2 Z_{11b}(\omega)^3} \tag{3}$$

$$g_1 = n_1 g_2 \tag{4}$$

$$c_1 = n_2 g_2 \tag{5}$$

$$c_2 = n_3 g_2$$

$$Z = \frac{\frac{1}{Z_{12}} - g_1 - g_2 - j\omega c_1 - j\omega c_2}{(g_1 + j\omega c_1)(g_2 + j\omega c_2)} \tag{6}$$

where ω is test frequency, Z_{ija} and Z_{ijb}, $i=1,2$, $j=1,2$ are the real and imaginary parts of the Z-parameters respectively, and n_k, $k=1,2,3$ can be calculated as follows,

$$n_1 = -\frac{a_1 a_2 b_5 \omega\omega_2 + b_1 b_4 b_5 - (b_3)^2 b_4 + a_2 a_3 b_1 \omega\omega_2 - a_1 a_3 b_3 \omega\omega_2 - (a_2)^2 b_3 \omega\omega_2}{\Delta_1} \tag{7}$$

$$n_2 = -\frac{a_1 b_4 b_5 + a_1 a_2 a_3 \omega\omega_2 - a_2 b_2 b_5 - (a_2)^3 \omega\omega_2 - a_2 b_3 b_4 + a_2 b_2 b_3}{\Delta_1} \tag{8}$$

$$n_3 = \frac{-a_2 b_1 b_4 + a_3 b_1 b_2 - a_1 (a_2)^2 \omega\omega_2 - a_2 b_2 b_3 + (a_1)^2 a_3 \omega\omega_2 + a_1 b_3 b_4}{\Delta_1} \tag{9}$$

where

$$\Delta_1 = b_1 b_2 b_5 + (a_1)^2 b_5 \omega\omega_2 - b_2 (b_3)^2 + (a_2)^2 b_1 \omega\omega_2 - 2 a_1 a_2 b_3 \omega\omega_2 \tag{10}$$

and a_k, $k=1,2,3$, b_j, $j=1\ldots6$ can be calculated directly from the knowns(See [1] for their definations)

One problem with this set of formula is that Δ_1 can be zero for some particular measurement values. In this case, an alternative has to be found to calculate the parameters. Taking into

consideration the condition that Δ_1 is zero, another set of formula can be obtained as shown in the follows.

$$r_1 = -\frac{-Z_{11a}\omega a_2 b_5 + Z_{11a}\omega a_3 b_3 - Z_{12b} b_1 b_5 + Z_{12b}(b_3)^2 - Z_{12a}\omega a_3 b_1 + Z_{12a}\omega a_2 b_3}{\omega(a_2 b_5 - a_3 b_3)} \tag{11}$$

$$r_2 = \frac{Z_{22a}\omega a_1 b_3 - Z_{22a}\omega a_2 b_1 - Z_{12b} b_1 b_5 + Z_{12b}(b_3)^2 - Z_{12a}\omega a_1 b_5 + Z_{12a}\omega a_2 b_3}{\omega(a_1 b_3 - a_2 b_1)} \tag{11}$$

$$g_2 = \frac{s_2 - m_2}{m_1 s_2 - m_2 s_1} \tag{13}$$

$$c_2 = \frac{m_1 - s_1}{m_1 s_2 - m_2 s_1} \tag{14}$$

$$c_1 = q_1 c_2 + q_2 g_2 \tag{15}$$

$$c_1 = p_1 c_2 + p_2 g_2 \tag{16}$$

where

$$m_1 = Z_{11a} p_2 - \omega Z_{11b} q_2 - r_1 p_2 + Z_{12a} \tag{17}$$

$$m_2 = Z_{11a} p_1 - \omega Z_{11b} q_1 - r_1 p_1 + \omega Z_{12b} \tag{18}$$

$$s_1 = Z_{22a} - r_2 + Z_{12a} p_2 - \omega Z_{12b} q_2 \tag{19}$$

$$s_2 = -\omega Z_{22b} + Z_{12a} p_1 - \omega Z_{12b} q_1 \tag{20}$$

and

$$p_1 = \frac{(b_3)^2 - b_1 b_5}{a_1 b_3 - a_2 b_1} \tag{21}$$

$$p_2 = \frac{a_3 b_1 - a_2 b_3}{a_1 b_3 - a_2 b_1} \tag{22}$$

$$q_1 = \frac{a_2 b_3 - a_1 b_5}{a_1 b_3 - a_2 b_1} \tag{23}$$

$$q_2 = \frac{a_1 a_3 - (a_2)^2}{a_1 b_3 - a_2 b_1} \tag{24}$$